Linear Programming and Applications

Finite Math

21 March 2019

Quiz

What is the solution to a linear programming problem if the feasible region is empty?

When Can We Solve This?

Theorem (Existence of Optimal Solutions)

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(C) *If the feasible region is empty, then both the maximum value and the minimum value of the objective function do not exist.*

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3. *Construct a corner point table listing the value of the objective function at each corner point.*
4. *Determine the optimal solution(s) from the table in Step 2 (smallest=minimum, largest=maximum).*
5. *For an applied problem, interpret the optimal solution(s) in terms of the original problem.*

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Solution

Minimum of $z = 14$ *at* $(4, 2)$. *No maximum.*

Now You Try It!

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Maximize and minimize $P = 30x + 10y$ *subject to*

$$\begin{aligned} 2x + 2y &\geq 4 \\ 6x + 4y &\leq 36 \\ 2x + y &\leq 10 \\ x, y &\geq 0 \end{aligned}$$

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Solution

Minimum of $P = 20$ *at* $(0, 2)$. *Maximum of* $P = 150$ *at* $(5, 0)$.

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Maximize and minimize $P = 3x + 5y$ *subject to*

$$\begin{aligned} x + 2y &\leq 6 \\ x + y &\leq 4 \\ 2x + 3y &\geq 12 \\ x, y &\geq 0 \end{aligned}$$

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Example

Maximize and minimize $P = 3x + 5y$ *subject to*

$$\begin{aligned} x + 2y &\leq 6 \\ x + y &\leq 4 \\ 2x + 3y &\geq 12 \\ x, y &\geq 0 \end{aligned}$$

Solution

No optimal solutions.

Applications

Example

An electronics firm manufactures two types of personal computers–a desktop model and a laptop model. The production of a desktop requires a capital expenditure of $400 *and* 40 *hours of labor. The production of a laptop requires a capital expenditure of* $250 *and* 30 *hours of labor. The firm has* $20,000 *capital and* 2,160 *labor-hours available for production of desktop and laptop computers.*

(a) *What is the maximum number of computers the company is capable of producing?*

(b) *If each desktop contributes a profit of* $320 *and each laptop contributes a profit of* $220*, how much profit will the company make by producing the maximum number of computers?*

(c) *Does producing as many computers as possible produce the highest profit? If not, what is the highest profit and how many of each computer should be made in that case?*

Now You Try It!

Example

A fruit grower can use two types of fertilizer in his orange grove, brand A and brand B. The amounts (in pounds) of nitrogen, phosphoric acid, and chloride in a bag of each brand are given in the table. Tests indicate that the grove needs at least 1,000 *pounds of phosphoric acid and at most* 400 *pounds of chloride.*

	Brand A	*Brand B*
Nitrogen	8	3
Phosphoric Acid	4	4
Chloride	2	1

(a) *If the grower wants to maximize the amount of nitrogen added to the grove, how many bags of each mix should be used? How much nitrogen will be added?*

(b) *If the grower wants to minimize the amount of nitrogen added to the grove, how many bags of each mix should be used? How much nitrogen will be added?*

Answer

Solution

(a) 150 *bags brand A*, 100 *bags brand B*, $1,500$ *lbs of nitrogen*

(b) 0 *bags brand A*, 250 *bags brand B*, 750 *lbs of nitrogen*